

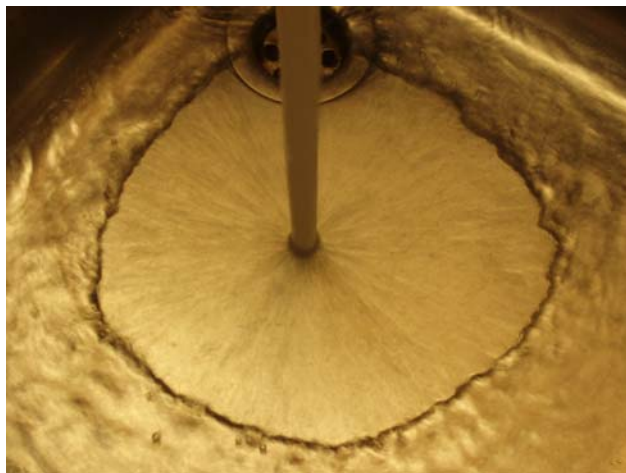
The Momentum Principle



Hydromechanics VVR090



Hydraulic Jump



(photo taken 2008-02-24 in M. Larson's kitchen sink)



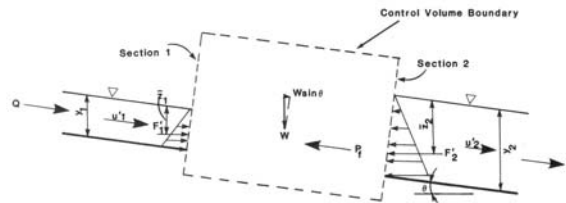
Specific Momentum

Horizontal momentum equation is applied to a control volume in an open channel (x-direction):

$$F_1' + F_3' - F_2' - P_f = \frac{\gamma}{g} Q (\beta_2 \bar{u}_2' - \beta_1 \bar{u}_1')$$

Small slopes:

$$\gamma \bar{z}_1 A_1 - \gamma \bar{z}_2 A_2 - P_f = \frac{\gamma}{g} Q (\bar{u}_2 - \bar{u}_1)$$



Continuity equation:

$$\bar{u}_1 = \frac{Q}{A_1}, \quad \bar{u}_2 = \frac{Q}{A_2}$$

Rearranging the momentum equation:

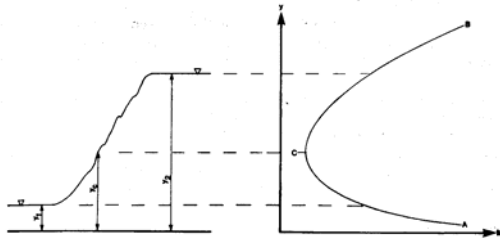
$$\frac{P_f}{\gamma} = \left(\frac{Q^2}{gA_1} + \bar{z}_1 A_1 \right) - \left(\frac{Q^2}{gA_2} + \bar{z}_2 A_2 \right)$$

$$\frac{P_f}{\gamma} = M_1 - M_2$$

$$M = \frac{Q^2}{gA} + \bar{z}A$$



Specific Momentum Curve



Analogous to the specific energy curve.

For a given value on M there are in general two possible water depths (**sequent depths** of a hydraulic jump).

The minimum value of M corresponds to the minimum value of E .



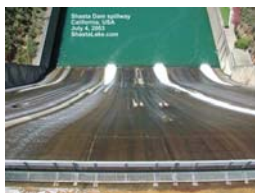
The Hydraulic Jump

A hydraulic jump results when there is a conflict between upstream and downstream control.

Example: upstream control induce supercritical flow and downstream control subcritical flow.

=> flow has to pass from one regime to another creating a hydraulic jump

Significant turbulence and energy dissipation occurs in a hydraulic jump.



Applications of the Hydraulic Jump

1. Dissipation of energy in flows over dams, weirs, and other hydraulic structures
2. Maintenance of high water levels in channels for water distribution
3. Increase of discharge of a sluice gate by repelling the downstream tailwater
4. Reduction of uplift pressure under structures by raising the water depth
5. Mixing of chemicals
6. Aeration of flow
7. Removal of air pockets
8. Flow measurements



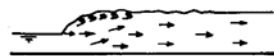
Hydraulic Jump Dependence on Froude Number



Vågformigt vattensprång
 $F = 1-1,7$



Instabilt vattensprång
 $F = 2,5-4,5$



Svagt vattensprång
 $F = 1,7-2,5$



Stationärt vattensprång
 $F = 4,5-9,0$



Kraftigt vattensprång
 $F > 9,0$



Hydraulic Jump in a Sink



Governing Equation for a Hydraulic Jump I

Apply the momentum equation for a finite control volume encompassing the jump and let $P_f = 0$:

$$M_1 = M_2$$

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$$

For a rectangular section:

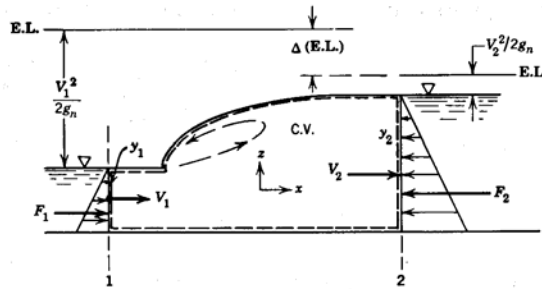
$$Q = \bar{u}_1 A_1 = \bar{u}_2 A_2$$

$$A_1 = by_1, \quad A_2 = by_2$$

$$\bar{z}_1 = y_1/2, \quad \bar{z}_2 = y_2/2$$



Governing Equation for a Hydraulic Jump II



Momentum equation (rectangular channel):

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} = \frac{\gamma q}{g} (\bar{u}_2 - \bar{u}_1)$$



Momentum equation for rectangular section:

$$\frac{q^2}{g} \left(\frac{1}{y_1} - \frac{1}{y_2} \right) = \frac{1}{2} (y_2^2 - y_1^2)$$

Solutions:

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8Fr_1^2} - 1 \right)$$

$$\frac{y_1}{y_2} = \frac{1}{2} \left(\sqrt{1 + 8Fr_2^2} - 1 \right)$$



Downstream Froude number (Fr_2) usually small so:

$$\sqrt{1+8Fr_2^2} - 1 \approx 0$$

Sometimes useful to do a Taylor series expansion:

$$\sqrt{1+8Fr_2^2} = 1 + 4Fr_2^2 - 8Fr_2^4 + 32Fr_2^6 + \dots$$

$$\left(\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots \right)$$

Substituting into the hydraulic jump equation:

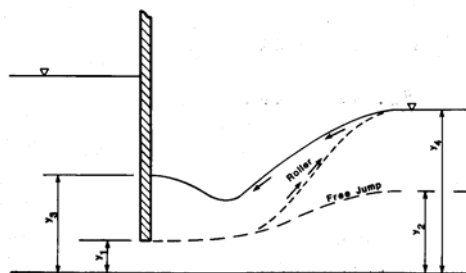
$$\frac{y_1}{y_2} = 2Fr_2^2 - 4Fr_2^4 + 16Fr_2^6 + \dots$$



Submerged Jump

If the downstream water level is high enough ($>$ sequent depth y_2) the jump might be submerged. The depth of submergence (y_3) is then a crucial unknown.

Could occur downstream gates etc.



Submergence of Hydraulic Jump



Govinda (1963) proposed the following formula for submerged jumps (horizontal, rectangular channels):

$$\frac{y_3}{y_1} = \left[(1+S)^2 \phi^2 - 2Fr_1^2 + \frac{2Fr_1^2}{(1+S)\phi} \right]^{1/2}$$

$$S = \frac{y_4 - y_1}{y_2}$$

$$\phi = \frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8Fr_1^2} - 1 \right)$$

Chow (1959):
$$\frac{y_3}{y_4} = \left[1 + 2Fr_4^2 \left(1 - \frac{y_4}{y_1} \right) \right]^{1/2}$$



Energy Loss in a Hydraulic Jump I

Dissipation in a horizontal channel:

$$\Delta E = E_1 - E_2$$

Rectangular section:

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\frac{\Delta E}{E_1} = \frac{2 - 2(y_2/y_1) + Fr_1^2 \left[1 - (y_1/y_2)^2 \right]}{2 + Fr_1^2}$$

$$\frac{E_1}{E_2} = \frac{(8Fr_1^2 + 1)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2 + Fr_1^2)}$$



Energy Loss in a Hydraulic Jump II

Energy equation across the jump:

$$y_1 + \frac{u_1^2}{2g} = y_2 + \frac{u_2^2}{2g} + \Delta E$$

$$\Delta E = \frac{u_1^2}{2g} \left(1 - \frac{y_1^2}{y_2^2} \right) + y_1 - y_2$$

$$\Delta E = \frac{y_2 - y_1}{4y_2y_1} \left[(y_2 + y_1)^2 - 4y_1y_2 \right]$$

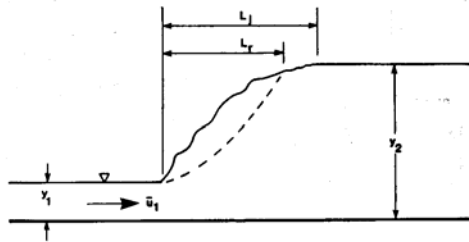
$$\longrightarrow \Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$



Hydraulic Jump Length I

Jump length hard to derive through theoretical considerations.

Length of the jump (L_j) is defined as the distance from the front face of the jump to a point on the surface immediately downstream the roller associated with the jump.



Hydraulic Jump Length II

Empirical equation for jump length given by Silvester (1964) for rectangular channels,

$$\frac{L_j}{y_1} = 9.75 (Fr_1 - 1)^{1.01}$$

and for triangular channels:

$$\frac{L_j}{y_1} = 4.26 (Fr_1 - 1)^{0.695}$$

Submerged jump: $\frac{L_j}{y_2} = 4.9S + 6.1$



Hydraulic Jump in Sloping Channels

A number of different cases have to be considered.

