Non-Uniform Flow

1. (9.7 extra)

a.

The water depth in the channel is constant $y = y_n = 1.5$ m, which implies that uniform flow prevails in the channel. The flow rate is calculated using the Manning formula:

$$Q = \frac{1}{n} AR^{2/3} S_o^{1/2}$$

The flow area, the wetted perimeter, and the hydraulic radius is determined to be (following the formulas in French page 12):

$$A = (10 + 2y_n)y_n$$

$$P = 10 + 2y_n\sqrt{1 + 2^2}$$

$$R = \frac{A}{P} = \frac{(10 + 2y_n)y_n}{10 + 2y_n\sqrt{5}}$$

which yields for $y_n = 1.5$ m, $A = 19.5$ m$^2$, $P = 16.7$ m, and $R = 1.17$ m

Enter $A$ and $R$ into the Manning equation:

$$Q = \frac{1}{0.033} 19.5 \cdot 1.17^{2/3} \cdot \left( \frac{1}{2000} \right)^{1/2} = 14.5 \text{ m}^3/\text{s}$$

b.

If the water depth is constant in the channel and equal to the normal depth ($y_n$), the flow has to be subcritical and the depth is $y_n$ in the inflow cross section at the upper lake. This can be further checked by calculating the Froude number. First, the mean velocity is given by:

$$u_n = \frac{Q}{A} = \frac{14.5}{19.5} = 0.74 \text{ m/s}$$
The Froude number is obtained from:

\[ Fr = \frac{u_n}{\sqrt{gA/T}} = \frac{0.74\sqrt{10 + 2 \cdot 1.5 \cdot 2}}{\sqrt{9.81 \cdot 19.5}} = 0.21 < 1 \]

Since \( Fr < 1 \), the flow is subcritical in the channel.

In order to determine the water level of the upper lake, apply the energy equation between the upper lake and the inflow cross section (A),

\[ H = 13.0 + y_n + \alpha \frac{u_n^2}{2g} + k_n \frac{u_n^2}{2g} \]

\[ H = 13.0 + 1.5 + 1.15 \frac{0.74^2}{2g} + 0.2 \frac{0.74^2}{2g} = 14.54 \text{ m} \]

where \( \alpha = 1.15 \) was used.

c.

The depth change in the channel should be investigated when the water level in the downstream lake increases to +12.0 m. If this water level increase does not affect conditions at A, the flow will be the same as before – otherwise the flow rate will decrease. The former situation is assumed here.

The detailed shape of the water surface profile upstream the lake is determined through a step calculation, starting at the control point and proceeding upstream, against the flow (subcritical flow conditions).

The energy equation between two locations \( i \) and \( i+1 \), including friction, is given by,

\[ y_i + \alpha \frac{u_i^2}{2g} + z_i = y_{i+1} + \alpha \frac{u_{i+1}^2}{2g} + z_{i+1} + h_{li,i+1} \]

where \( h_{li,i+1} \) is the energy loss between \( i \) and \( i+1 \). Introducing the slope of the channel \( (S_o) \) and energy grade line \( (S_f) \) into the energy equation gives:
\[ y_i + \frac{\alpha u_i^2}{2g} - y_{i+1} - \frac{\alpha u_{i+1}^2}{2g} = S_f \Delta x - S_o \Delta x \]

\[ \Delta x = \frac{y_i + \frac{\alpha u_i^2}{2g} - y_{i+1} - \frac{\alpha u_{i+1}^2}{2g}}{S_f - S_o} \]

The mean energy loss is estimated from (note: this is done somewhat differently from previous calculations):

\[ S_f = \frac{n^2 \bar{u}^2}{R^{5/3}} \]

\[ \bar{u} = \frac{Q}{0.5(A_i + A_{i+1})} \]

\[ \bar{R} = \frac{A_i + A_{i+1}}{P_i + P_{i+1}} \]

Make a table to perform the step calculation, starting at the appropriate water depth and move upstream the distance of interest. Use suitable steps in the depth.

<table>
<thead>
<tr>
<th>( y_i )</th>
<th>( A_i )</th>
<th>( P_i )</th>
<th>( \bar{u}_i )</th>
<th>( \bar{R}_i )</th>
<th>( S_f )</th>
<th>( u_i^2/2g )</th>
<th>( \Delta y )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>28.00</td>
<td>18.94</td>
<td></td>
<td></td>
<td></td>
<td>0.0137</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1.90</td>
<td>26.22</td>
<td>18.50</td>
<td>0.535</td>
<td>1.448</td>
<td>0.194 \times 10^{-3}</td>
<td>0.0156</td>
<td>320</td>
<td>320</td>
</tr>
<tr>
<td>1.80</td>
<td>24.48</td>
<td>18.05</td>
<td>0.572</td>
<td>1.387</td>
<td>0.235 \times 10^{-3}</td>
<td>0.0179</td>
<td>369</td>
<td>689</td>
</tr>
<tr>
<td>1.70</td>
<td>22.78</td>
<td>17.60</td>
<td>0.614</td>
<td>1.326</td>
<td>0.288 \times 10^{-3}</td>
<td>0.0206</td>
<td>458</td>
<td>1147</td>
</tr>
<tr>
<td>1.60</td>
<td>21.12</td>
<td>17.16</td>
<td>0.661</td>
<td>1.263</td>
<td>0.356 \times 10^{-3}</td>
<td>0.0240</td>
<td>669</td>
<td>1861</td>
</tr>
<tr>
<td>1.55</td>
<td>20.31</td>
<td>16.93</td>
<td>0.700</td>
<td>1.215</td>
<td>0.420 \times 10^{-3}</td>
<td>0.0260</td>
<td>599</td>
<td>2415</td>
</tr>
<tr>
<td>1.50</td>
<td>19.50</td>
<td>16.71</td>
<td>0.728</td>
<td>1.183</td>
<td>0.471 \times 10^{-3}</td>
<td>0.0282</td>
<td>1631</td>
<td>4046</td>
</tr>
</tbody>
</table>

The uniform depth is obtained about 4100 m upstream in the channel. Thus, the assumption made that the water level increase in the downstream lake does not affect the flow is correct (4100 m < 6000 m).

2. **(14.326 extra)**

The water is discharged through a “free overfall” where the channel flow accelerates down a slope and falls freely over a sharp edge. The flow becomes critical just before the
overfall; however, this is assumed to occur approximately at the edge. The flow rate is \( q = 1.1 \text{ m}^{3}/\text{m/s} \) (per unit width), implying that the critical depth is:

\[
y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{1.1^2}{g} \right)^{1/3} = 0.50 \text{ m}
\]

Far upstream in the channel the normal water depth is attained, determined from the Manning equation as:

\[
q = \frac{1}{n} y_n^{2/3} S_o^{1/2}
\]

\[
1.1 = \frac{1}{0.014} y_n^{5/3} 0.001047^{1/2},
\]

\[
\rightarrow y_n = 0.64 \text{ m}
\]

Thus, \( y_n = 0.64 \text{ m} \) implies subcritical flow, as expected.

Since \( y_n > y > y_c \), an M2 water surface profile will form.

Make a table to perform the step calculation (for governing equations, see previous problem), starting at the appropriate water depth and move upstream the distance of interest. Use a step size in the depth of 0.02 m.

<table>
<thead>
<tr>
<th>( y_i )</th>
<th>( u_i )</th>
<th>( E = y_i/2g )</th>
<th>( S_{ii} )</th>
<th>( S_f )</th>
<th>( \Delta S )</th>
<th>( \Delta x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>2.20</td>
<td>0.74669</td>
<td>2.39 \times 10^{-1}</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0.52</td>
<td>2.12</td>
<td>0.74808</td>
<td>2.10 \times 10^{-3}</td>
<td>2.24 \times 10^{-3}</td>
<td>0.010</td>
<td>1.2</td>
</tr>
<tr>
<td>0.54</td>
<td>2.04</td>
<td>0.75149</td>
<td>1.85 \times 10^{-4}</td>
<td>1.97 \times 10^{-3}</td>
<td>0.028</td>
<td>3.7</td>
</tr>
<tr>
<td>0.56</td>
<td>1.96</td>
<td>0.75666</td>
<td>1.64 \times 10^{-4}</td>
<td>1.74 \times 10^{-3}</td>
<td>0.038</td>
<td>7.4</td>
</tr>
<tr>
<td>0.58</td>
<td>1.90</td>
<td>0.76333</td>
<td>1.46 \times 10^{-4}</td>
<td>1.55 \times 10^{-3}</td>
<td>0.048</td>
<td>13.3</td>
</tr>
<tr>
<td>0.60</td>
<td>1.83</td>
<td>0.77131</td>
<td>1.30 \times 10^{-4}</td>
<td>1.38 \times 10^{-3}</td>
<td>0.117</td>
<td>24.0</td>
</tr>
<tr>
<td>0.62</td>
<td>1.77</td>
<td>0.78044</td>
<td>1.17 \times 10^{-4}</td>
<td>1.23 \times 10^{-3}</td>
<td>0.139</td>
<td>48.0</td>
</tr>
<tr>
<td>0.64</td>
<td>1.72</td>
<td>0.79056</td>
<td>1.05 \times 10^{-4}</td>
<td>1.11 \times 10^{-3}</td>
<td>0.154</td>
<td>160.0</td>
</tr>
</tbody>
</table>

A water depth of \( y = 0.64 \text{ m} \) is obtained approximately 300 m upstream the edge.