1.

The excess temperature (i.e., the difference between the temperature in the jet and the ambient) is regarded as a concentration. The centerline concentration in a pure jet is given by:

\[ c_m = 5.6c_o \frac{D_o}{x} \]

Inserting numbers yields:

\[ 2 = 5.6 \cdot 10 \frac{D_o}{40} \]
\[ \rightarrow \quad D_o = 1.43 \text{ m} \]

The centerline velocity is obtained from:

\[ u_m = 6.2u_o \frac{D_o}{x} \]

Thus, the required exit velocity is:

\[ u_o = \frac{0.2 \cdot 40}{6.2 \cdot 1.43} = 0.9 \text{ m/s} \]

The flow obtained through this exit velocity for one nozzle (opening) is:

\[ Q_{on} = \frac{\pi D_o^2}{4} u_o = 1.45 \text{ m}^3/\text{s} \]

Thus, the total number of nozzles needed to achieve the desired discharge is:

\[ n = \frac{Q_o}{Q_{on}} = \frac{20}{1.45} = 13.8 \quad \Rightarrow \quad \text{select 14 nozzles} \]
2.

The dilution rate may be defined as:

\[ S_m = \frac{\rho_r - \rho_o}{\rho_r - \rho_m} = \frac{c_o}{c_m} \]

Additional mixing at the surface yields:

\[ \frac{c_m}{c} = 1.4 \]

Thus, the required dilution rate at the surface is (required density difference after mixing at surface is 0.3 kg/m\(^3\)):

\[ S_m = \frac{c_o}{c_m} = \frac{c_o}{1.4c} = \frac{\rho_r - \rho_o}{1.4(\rho_r - \rho_{after})} = \frac{1030 - 1005}{1.4 \cdot 0.3} = 59.5 \]

The exit velocity is given by:

\[ u_o = \frac{Q_o}{A} = \frac{Q_o}{\pi D_o^2} = \frac{0.040 \cdot 4}{\pi \cdot 0.15^2} = 2.26 \text{ m/s} \]

The densimetric Froude number is calculated from:

\[ F_D = \frac{u_o}{\sqrt{g \left( \frac{\rho_r - \rho_o}{\rho_o} D_o \right)}} = \frac{2.26}{\sqrt{9.81 \frac{1030 - 1005}{1005} \cdot 0.15}} = 11.8 \]

Use figure 40 (page 43) with \( S_m = 59.5 \) and \( F_D = 11.8 \), which gives:

\[ \frac{x}{D_o} = 55 \]

\[ \frac{z}{D_o} = 97.5 \]

The water depth at the exit is \( z = 97.5 \cdot 0.15 = 14.6 \text{ m} \)

The total pipe length for a bottom slope of 1:100 is \( 14.6 \cdot 100 = 1460 \text{ m} \)
3.

The concentration in the jet just below the mixing zone at the surface is:

\[ c_m = 5.6 c_o \frac{D}{x} = 5.6 c_o D \frac{10}{9 H} \]

Continuity yields that the mass flux of the pollutant from the jet exit is equal to the transport out from the mixing zone:

\[ c_o Q_o = c_R Q_R \]

where index \( o \) denotes conditions at the jet exit and index \( R \) after the mixing zone. No additional flow is assumed to be entrained in the mixing zone, so the flow rate out from the mixing zone is the same as the rate into the zone given by:

\[ Q_R = 0.32 Q_o \frac{x}{D} = 0.32 Q_o \frac{1}{10} \frac{9 H}{D} \]

Thus, the concentration in the outflow from the mixing zone is:

\[ c_R = \frac{c_o Q_o}{0.32 Q_o} \frac{9 H}{10} \frac{1}{D} \]

The ratio between the concentration after and before the mixing zone is:

\[ \frac{c_R}{c_m} = \frac{c_o Q_o}{0.32 Q_o} \frac{9 H}{10} \frac{1}{5.6 c_o D} = 0.56 \]

Thus, the additional dilution in the mixing zone close to the water surface is \( 1/0.56 = 1.8 \) (compare with the empirically based value we normally use of 1.4).

4.

Dilution rate at the jet axis immediately before mixing at the surface:

\[ S_m = \frac{\rho_r - \rho_o}{\rho_r - \rho_m} = \frac{c_o}{c_m} \]

Additional mixing at the surface yields:
\[ \frac{c_m}{c_{after}} = 1.4 \]

The density difference between the discharged water and the receiving water should be less than 0.5 kg/m\(^3\) after mixing at the surface. Thus, the density difference should be \(1.4 \cdot 0.5 = 0.7\) kg/m\(^3\) before the mixing zone. This corresponds to a required dilution rate of:

\[ S_m = \frac{1016.3 - 999}{0.7} \approx 25 \]

The diagram over jet trajectories on page 43 should be used. Since the diameter is unknown an iterative approach is needed: Guess \(z/D_o\), which gives \(D_o\) that can be used to compute the densimetric Froude number \(F_o\). If the computed \(F_o\) yields the assumed \(z/D_o\)-value the correct solution has been found.

Assume \(z/D_o = 50\) \(\Rightarrow\) \(D_o = 10/50 = 0.2\) m

\[ F_o = \sqrt{\frac{2}{g \frac{17.3}{999} \cdot 0.2}} = 11 \Rightarrow \text{Slightly lower value on } z/D_o \]

Try \(z/D_o = 45\) \(\Rightarrow\) \(D_o = 10/45 = 0.22\) m

\[ F_o = \sqrt{\frac{2}{g \frac{17.3}{999} \cdot 0.22}} = 10.3 \Rightarrow \text{OK} \]

Assume that \(n\) nozzles are needed, which implies a total discharge area of:

\[ A = n \pi \cdot \frac{0.22^2}{4} = 0.038n \]

A flow of \(Q_o = 0.30\) m\(^3\)/s should be discharged giving a required number of nozzles of:

\[ 0.30 = n \cdot 0.038 \cdot 2.0 \Rightarrow n = 4 \]

5.

Since \(c_A < c_C\), the centerline (with the maximum concentration value) must be located between B and C. Geometrical relationships:

\[ r_A = r_B + 1.5 \Rightarrow r_A^2 - r_B^2 = 3r_B + 1.5^2 \]
Centerline concentration:

\[ c_m = 5.6 c_o \frac{D_o}{x} \]

Concentration distribution in the jet (Gaussian):

\[ c = c_m \exp \left(-62 \frac{r^2}{x^2}\right) \]

This could be rewritten:

\[ -62 \frac{r^2}{x^2} = \ln \left( \frac{c}{c_m} \right) \]

\[ \Rightarrow \quad r^2 = -\frac{x^2}{62} \ln \left( \frac{c}{c_m} \right) = -\frac{x^2}{62} \ln \left( \frac{c}{5.6 c_o D_o} \right) = \]

\[ = \frac{x^2}{62} \left( \ln c_o - \ln \left( \frac{c}{5.6 D_o} \right) \right) \]

Substituting for the two points A and B gives:

\[ r_A^2 = \frac{x^2}{62} \left( \ln c_o - 3.98 \right) \]

\[ r_B^2 = \frac{x^2}{62} \left( \ln c_o - 4.60 \right) \]

Subtracting the second equation from the first yields:

\[ r_A^2 - r_B^2 = \frac{x^2}{62} (4.60 - 3.98) = 3.97 \]

Use earlier derived geometrical relationship:

\[ 3 r_B^2 + 1.5^2 = 3.97 \]

\[ \Rightarrow \quad r_B = 0.58, \quad r_A = 2.08 \]

The centerline concentration is obtained from:
Finally, the concentration at the jet exit is:

\[ c_o = \frac{c_m x}{5.6 D_o} = 104.4 \text{ mg/l} \]

6.

Calculate for the maximum temperature difference, which gives maximum density difference and minimum densimetric Froude number.

Velocity at jet exit:

\[ u_o = \frac{Q}{A} = \frac{0.24 \cdot 4}{\pi \cdot 0.6^2} = 0.89 \text{ m/s} \]

Densimetric Froude number:

\[ F_o = \frac{u_o}{\sqrt{\frac{\rho_r - \rho_o}{\rho_o} g D_o}} = \frac{0.89}{\sqrt{\frac{0.99995 - 0.999}{0.999} g \cdot 0.6}} \approx 12 \]

Required dilution is:

\[ S_m = \frac{c_o}{c_m} = \frac{1}{0.2} = 5 \]

Use diagram from Assignment 1 (diagram on page 43 has poor resolution in this region):

\[ S_m = 5, \quad F_o = 12 \quad \Rightarrow \quad \frac{z}{D_o} = 8 \]

\[ z = 8 \cdot 0.6 \approx 5 \text{ m} \]

7.

Jet velocity at exit:

\[ u_o = \frac{Q}{A} = \frac{0.022 \cdot 4}{\pi \cdot 0.15^2} = 1.24 \text{ m/s} \]
Density difference between the receiving water and the discharge in the lower layer is $\Delta \rho = 1010 - 998 = 12 \text{ kg/m}^3$ at jet exit. The densimetric Froude number is:

$$F_o = \frac{u_o}{\sqrt{\Delta \rho g D_o \rho_o}} = \frac{1.24}{\sqrt{12 \cdot \frac{g}{998} \cdot 0.15}} = 9.3$$

The jet will most likely be trapped in the transition zone. It will for sure pass the lower layer since the entrained higher density water from the ambient can not increase the jet density to 1010 kg/m$^3$. In order to study the jet trajectory calculations are done at 1-m steps through the vertical until the density of the jet becomes equal to the density of the ambient (trapping will occur then). It is suitable to make a table according to:

<table>
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<tr>
<th>Water depth</th>
<th>$z/D_o$</th>
<th>$S_m$</th>
<th>$\rho_m$</th>
<th>$\rho_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0</td>
<td>1</td>
<td>998</td>
<td>1010</td>
</tr>
<tr>
<td>14</td>
<td>6.7</td>
<td>5</td>
<td>1007.6</td>
<td>1010</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>7</td>
<td>1008.3</td>
<td>1010</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>10</td>
<td>1008.8</td>
<td>1010</td>
</tr>
<tr>
<td>11</td>
<td>27</td>
<td>15</td>
<td>1009.2</td>
<td>1010</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
<td>17</td>
<td>1009.3</td>
<td>1010</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>21</td>
<td>1009.4**</td>
<td>1008</td>
</tr>
</tbody>
</table>

* is obtained as $1/5 \cdot 998 + 4/5 \cdot 1010 = 1007.6$

** is obtained as $1/21 \cdot 998 + 20/21 \cdot 1010 = 1009.4$

$\rho_m = \rho_r$ at water depth 9.7 m where the jet will be trapped. The dilution at this depth is about 18 times (linear interpolation)